

On effective actions of non-BPS branes and their higher derivative corrections

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Abstract

By calculating various disk level S-matrix elements and studying in details their momentum expansions, we have extracted some of the couplings in tachyon DBI action and Wess-Zumino terms of the non-BPS branes, and their higher derivative corrections. In particular, we have found that there is exact consistency between field theory and string theory tachyon pole of S-matrix element of one RR and three tachyons provided that one takes into account the fact that the tachyon vertex operator in 0 picture to be along the Pauli matrix σ_1 whereas the tachyon in -1 picture to be along the σ_2 direction. This internal CP factors should be included in the tachyon DBI part of the effective action.

1 Introduction

Study of unstable objects in string theory might shed new light in understanding properties of string theory in time-dependent backgrounds [1, 2, 3, 4, 5, 6]. The source of instability is appearance of some tachyon modes in the spectrum of these objects. It then makes sense to study them in a field theory which includes the massless and tachyon fields. In this regard, it has been shown by A. Sen that an effective action of Born-Infeld type proposed in [7, 8, 9, 10] can capture many properties of the decay of non-BPS D_p -branes in string theory [2, 3].

The non-BPS D_p -branes of type IIA(B) string theory are defined by projecting D_p -brane-anti- D_p -brane of type IIB(A) with $(-1)^{F_L}$ where F_L denotes the contribution to the space-time fermion number from the left-moving sector of the string world-sheet [11]. The open strings of the brane-anti-brane can be labeled by the *external* 2×2 Chan-Paton factors:

$$(a) : \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad (b) : \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad (c) : \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad (d) : \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

The massless states carry CP factor (a), (b), and the tachyons carry (c) and (d) factors. The projection operator $(-1)^{F_L}$ has no effect on the world-sheet fields, however, using the fact that it exchanges brane with anti-brane, one observes that its effect on the CP matrix Λ is the following:

$$\Lambda \rightarrow \sigma_1 \Lambda (\sigma_1)^{-1},$$

where σ_1 is the Pauli matrix. The states with CP factors I and σ_1 are survived. The massless fields then carry the *internal* CP factor I and the real tachyon of non-BPS D -brane carries the CP factor σ_1 . The vertex operators corresponding to the above states appear in S-matrix elements in different pictures. We assign the above internal CP factors to the vertex operators in the 0 picture. Using the observation made in [12] that the picture changing operator on a non-BPS brane naturally comes with σ_3 , one observes that in -1 picture, the internal CP factor of massless states is σ_3 and the CP factor of tachyon is σ_2 .

Another interesting object in string theory is SD_p -brane. In field theory, as the kink solution of the non-BPS D -brane effective action gives BPS D -brane, the kink solution in the time direction gives S-brane. In string theory, it is defined as an object on which open strings with Dirichlet boundary condition on the first $9-p$ directions X^a , $a = 0, 1, \dots, 8-p$ and Neumann boundary condition on the last $p+1$ coordinates X^i , $i = 9-p, \dots, 9$ end [1]. The SD_p -branes in Type II theory carry the RR charges, and the non-BPS SD_p -branes

of type IIA(B) string theory are defined by projecting SD_p -brane-anti- SD_p -brane of type IIB(A) with $(-1)^{F_L}$.

The effective action of a non-BPS brane/brane-anti-brane has two parts, the part which is an extension of DBI action and the Wess-Zumino part, *i.e.*,

$$S_{non-BPS} = S_{DBI} + S_{WZ}$$

The WZ term describing the coupling of RR field to the gauge field of brane-anti-brane is given by [17, 18]

$$S = \mu_p \int_{\Sigma_{(p+1)}} C \wedge \text{Tr} \left(e^{i2\pi\alpha' F^{(1)}} - e^{i2\pi\alpha' F^{(2)}} \right), \quad (1)$$

where $\Sigma_{(p+1)}$ is the world volume and μ_p is the RR charge of the branes. In above equation, C is a formal sum of the RR potentials $C = \sum_{m=p+1} (-i)^{\frac{p+1-m}{2}} C_m$. Note that the factors of i disappear in each term of (1). The inclusion of the tachyon fields into this action has been proposed in [19, 20, 21] using the superconnection of noncommutative geometry [22, 23, 24]

$$S_{WZ} = \mu_p \int_{\Sigma_{(p+1)}} C \wedge \text{STr} e^{i2\pi\alpha' \mathcal{F}} \quad (2)$$

where the “supertrace” is defined as:

$$\text{STr} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \text{Tr} A - \text{Tr} D.$$

and the curvature of the superconnection is defined as:

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A} \wedge \mathcal{A}$$

the superconnection is

$$i\mathcal{A} = \begin{pmatrix} iA^{(1)} & \beta T^* \\ \beta T & iA^{(2)} \end{pmatrix},$$

where β is a normalization constant with dimension $1/\sqrt{\alpha'}$. If one uses the standard non-abelian kinetic term for the dynamics of the tachyon field then the normalization of tachyon in the WZ action (2) has to be [25]

$$\beta = \frac{1}{\pi} \sqrt{\frac{2 \ln(2)}{\alpha'}} \quad (3)$$

The WZ terms of the non-BPS branes can also be written in terms of superconnection [20, 21],

$$S_{WZ} = \mu'_p \int_{\Sigma_{(p+1)}} C \wedge \text{Str} e^{i2\pi\alpha' \mathcal{F}} \quad (4)$$

where the “supertrace” is now

$$\text{Str} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \text{Tr } B + \text{Tr } C .$$

and the superconnection is

$$i\mathcal{A} = \begin{pmatrix} iA & \beta'T \\ \beta'T & iA \end{pmatrix} ,$$

where β' is a new normalization constant with dimension $1/\sqrt{\alpha'}$. Using the multiplication rule of the supermatrices [20]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} AA' + (-)^{c'} BC' & AB' + (-)^{d'} BD' \\ DC' + (-)^{a'} CA' & DD' + (-)^{b'} CB' \end{pmatrix}$$

where x' is 0 if X is an even form or 1 if X is an odd form, one finds that the curvature of the superconnection is

$$i\mathcal{F} = \begin{pmatrix} iF - \beta'^2 T^2 & \beta' DT \\ \beta' DT & iF - \beta'^2 T^2 \end{pmatrix} ,$$

where $F = \frac{1}{2} F_{ab} dx^a \wedge dx^b$ and $DT = (\partial_a T - i[A_a, T]) dx^a$. In equation (4), C is a formal sum of the RR potentials $C = \sum_{m=p} (-i)^{\frac{p-m}{2}} C_m$. Using the expansion for the exponential term in the WZ action (4), one finds the following terms:

$$\begin{aligned} \mu'_p (2\pi\alpha') C \wedge \text{Str } i\mathcal{F} &= 2\beta' \mu'_p (2\pi\alpha') \text{Tr} (C_p \wedge DT) \\ \frac{\mu'_p}{2!} (2\pi\alpha')^2 C \wedge \text{Str } i\mathcal{F} \wedge i\mathcal{F} &= 2\beta' \mu'_p (2\pi\alpha')^2 \text{Tr} \left(-\beta'^2 T^2 C_p \wedge DT + C_{p-2} \wedge DT \wedge F \right) \\ \frac{\mu'_p}{3!} (2\pi\alpha')^3 C \wedge \text{Str } i\mathcal{F} \wedge i\mathcal{F} \wedge i\mathcal{F} &= \beta' \mu'_p (2\pi\alpha')^3 \text{Tr} \left(\beta'^4 T^4 C_p \wedge DT + C_{p-4} \wedge F \wedge F \wedge DT \right. \\ &\quad \left. + C_{p-2} \wedge \left(2\beta'^2 T^2 F \wedge DT + i \frac{\beta'^2}{3} DT \wedge DT \wedge DT \right) \right) \end{aligned} \tag{5}$$

where the trace is over the $U(N)$ matrices.

We shall confirm the above couplings with the S-matrix method, and find their higher derivative corrections. The S-matrix method can not be used to confirm the above WZ couplings in the non-BPS D-branes due to some kinematic reason [26]. However, there is no such kinematic obstacle in the non-BPS SD-brane case. To see this consider the coupling $C \wedge dT$. In the momentum space it is proportional to the tachyon momentum k . The conservation of momentum along the world-volume implies that $k^2 = p_i p^i$ where p_i is the RR momentum along the world-volume. Using the on-shell condition $m^2 = -k^2 = -1/4$

for tachyon¹ and $p_i p^i = -p_a p^a$ for the massless RR field, one finds that the RR momentum along the brane must be non-zero, *i.e.*, $p_i p^i = 1/4$ which is consistent with $p_i p^i = -p_a p^a$ as $p_a p^a$ can be negative for non-BPS SD-brane. For the non-BPS D-brane, on the other hand, the closed string on-shell condition is $p_i p^i = -p_a p^a < 0$ which is not consistent with $p_i p^i = 1/4$. Hence, the S-matrix method gives only the WZ couplings of non-BPS SD-branes.

An outline of the rest of paper is as follows. In the next section, to fix our notation we calculate the S-matrix element of one RR and one tachyon [19]. This gives the coupling $C_p \wedge DT$ in (4). In section 3, we calculate the S-matrix element of one RR, one tachyon and one gauge field. The momentum expansion of this amplitude gives the coupling $C_{p-2} \wedge DT \wedge F$ in (4) and its higher derivative corrections. In section 4, we calculate the S-matrix element of one RR and three tachyons and find its momentum expansion. This amplitude has two parts. The first part which involves C_{p-2} has massless pole and contact terms. The massless poles are reproduced by the higher derivative couplings of $C_{p-2} \wedge DT \wedge F$ and the standard non-abelian kinetic term of tachyon. The contact terms give coupling $C_{p-2} \wedge DT \wedge DT \wedge DT$ in (4) and its higher derivative corrections. The second part which involve C_p has tachyon pole and contact terms. The contact terms give the coupling $T^2 C_p \wedge DT$ and its higher derivative corrections. To produce the tachyon poles, one needs to know the couplings of four tachyons. In section 4.2.1, using the fact that at disk level two tachyons must carry σ_1 and the other two tachyons must carry the internal CP factor σ_2 , we calculate the S-matrix element of four tachyons from which we will find a bunch of higher derivative couplings of four tachyons. Using these couplings, we will show that the infinite tower of the tachyon poles of the S-matrix element of one RR and three tachyons are reproduced exactly by field theory. We discuss our results in section 5.

2 The $T - C$ amplitude

The two point amplitude between one RR and one tachyon is given by the following correlation functions:

$$\mathcal{A}^{T,RR} \sim \int dy d^2 z \langle V_T^{(-1)}(y) V_{RR}^{(-1)}(z, \bar{z}) \rangle \quad (6)$$

where the vertex operators are

$$V_T^{(-1)}(y) = e^{-\phi(y)} e^{2ik \cdot X(y)} \lambda \otimes \sigma_2 \quad (7)$$

¹Our convention sets $\alpha' = 2$.

$$V_{RR}^{(-1)}(z, \bar{z}) = (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} e^{-\phi(z)/2} S_\alpha(z) e^{ip \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{ip \cdot D \cdot X(\bar{z})} \otimes \sigma_3 \sigma_1$$

where k^i is the tachyon momentum and λ in the tachyon vertex operator is matrix in the $U(N)$ group. The RR vertex operator of brane-anti-brane system should carry the internal Pauli matrix σ_3 . To see this, we note that the σ -factor of the S-matrix element of one σ_1 -type tachyon, one σ_2 -type tachyon and one RR vertex operator of brane-anti-brane is non-zero which is consistent with the WZ terms of brane-anti-brane system [25]. Without the σ_3 factor the σ -factor of the S-matrix element is $\text{Tr}(\sigma_1 \sigma_2) = 0$ which is not consistent with the WZ couplings. For non-BPS branes, there should be an extra factor of σ_1 in the RR vertex operator. As argued in [11], since the RR state of non-BPS brane appears in the twisted sector when one regards type IIB as type IIA modded out by $(-1)^{F_L}$, there is a cut extending from RR vertex operator at the center of disk all the way to the boundary of the disk. At the point where the cut hits the boundary one needs to insert an extra factor of σ_1 .² The CP factor in the S-matrix element (6) is $\text{Tr}(\lambda) \otimes \text{Tr}(\sigma_2 \sigma_3 \sigma_1)$.

The projector in the RR vertex operator is $P_- = \frac{1}{2}(1 - \gamma^{11})$ and

$$\mathbb{H}_{(n)} = \frac{a_n}{n!} H_{\mu_1 \dots \mu_n} \gamma^{\mu_1} \dots \gamma^{\mu_n} ,$$

where $n = 2, 4$ for type IIA and $n = 1, 3, 5$ for type IIB. $a_n = i$ for IIA and $a_n = 1$ for IIB theory. The spinorial indices are raised with the charge conjugation matrix, *e.g.*, $(P_- \mathbb{H}_{(n)})^{\alpha\beta} = C^{\alpha\delta} (P_- \mathbb{H}_{(n)})_{\delta}{}^{\beta}$ (further conventions and notations for spinors can be found in appendix B of [27]). The RR bosons are massless so $p^2 = 0$ and for the tachyon $k^2 = 1/4$. The world-sheet fields have been extended to the entire complex plane. That is, we have replaced

$$\tilde{X}^\mu(\bar{z}) \rightarrow D_\nu^\mu X^\nu(\bar{z}) , \quad \tilde{\psi}^\mu(\bar{z}) \rightarrow D_\nu^\mu \psi^\nu(\bar{z}) , \quad \tilde{\phi}(\bar{z}) \rightarrow \phi(\bar{z}) , \quad \text{and} \quad \tilde{S}_\alpha(\bar{z}) \rightarrow M_\alpha{}^\beta S_\beta(\bar{z}) ,$$

where

$$D = \begin{pmatrix} -1_{9-p} & 0 \\ 0 & 1_{p+1} \end{pmatrix} , \quad \text{and} \quad M_p = \begin{cases} \frac{\pm i}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \dots \gamma^{i_{p+1}} \epsilon_{i_1 \dots i_{p+1}} & \text{for } p \text{ even} \\ \frac{\pm 1}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \dots \gamma^{i_{p+1}} \gamma_{11} \epsilon_{i_1 \dots i_{p+1}} & \text{for } p \text{ odd} \end{cases}$$

Using these replacements, one finds the standard propagators for the world-sheet fields X^μ, ϕ , *i.e.*,

$$\begin{aligned} \langle X^\mu(z) X^\nu(w) \rangle &= -\eta^{\mu\nu} \log(z - w) , \\ \langle \phi(z) \phi(w) \rangle &= -\log(z - w) . \end{aligned} \tag{8}$$

² In the conventions [11], the RR field which is in -2 picture comes with a single Pauli matrix σ_1 , in our conventions, it is in -1 picture hence it comes with $\sigma_3 \sigma_1$.

One also needs the correlation function between two spin operators. The correlation function involving an arbitrary number of ψ 's and two S 's is obtained using the following Wick-like rule [28]:

$$\begin{aligned} \langle \psi^{\mu_1}(y_1) \dots \psi^{\mu_n}(y_n) S_\alpha(z) S_\beta(\bar{z}) \rangle &= \frac{1}{2^{n/2}} \frac{(z - \bar{z})^{n/2-5/4}}{|y_1 - z| \dots |y_n - z|} \left[(\Gamma^{\mu_n \dots \mu_1} C^{-1})_{\alpha\beta} \right. \\ &\quad + \langle \psi^{\mu_1}(y_1) \psi^{\mu_2}(y_2) \rangle (\Gamma^{\mu_n \dots \mu_3} C^{-1})_{\alpha\beta} \pm \text{perms} \\ &\quad + \langle \psi^{\mu_1}(y_1) \psi^{\mu_2}(y_2) \rangle \langle \psi^{\mu_3}(y_3) \psi^{\mu_4}(y_4) \rangle (\Gamma^{\mu_n \dots \mu_5} C^{-1})_{\alpha\beta} \\ &\quad \left. \pm \text{perms} + \dots \right] \end{aligned} \quad (9)$$

where dots mean sum over all possible contractions. In above equation, $\Gamma^{\mu_n \dots \mu_1}$ is the totally antisymmetric combination of the gamma matrices and the Wick-like contraction, for real y_i , is given by

$$\langle \psi^\mu(y_1) \psi^\nu(y_2) \rangle = 2\eta^{\mu\nu} \frac{Re[(y_1 - z)(y_2 - \bar{z})]}{(y_1 - y_2)(z - \bar{z})}$$

The number of ψ in the correlators (6) is zero. Using the above formula for zero ψ and performing the other correlators using (8), one finds that the integrand is invariant under $SL(2, R)$ transformation. Gauge fixing this symmetry by fixing the position of vertex operators at $(x, y, z, \bar{z}) = (x, -x, i, -i)$, one finds [19]

$$\begin{aligned} \mathcal{A}^{T,RR} &\sim 2i \text{Tr}(P_- \not{H}_{(n)} M_p) \text{Tr}(\lambda) , \\ &= \left(\frac{i\pi\beta'\mu'_p}{2} \right) \text{Tr}(P_- \not{H}_{(n)} M_p) \text{Tr}(\lambda) . \end{aligned} \quad (10)$$

where the conservation of momentum along the world volume of brane, *i.e.*, $k^i + p^i = 0$, has been used. We have also normalized the amplitude by $\pi\beta'\mu'_p/4$.

The trace in (10) containing the factor of γ^{11} ensures the following results also hold for $p > 3$ with $H_{(n)} \equiv *H_{(10-n)}$ for $n \geq 5$. The trace is zero for $p+1 \neq n$, and for $n = p+1$ it is

$$\text{Tr}(\not{H}_{(n)} M_p) = \pm \frac{32}{(p+1)!} H_{i_1 \dots i_{p+1}} \epsilon^{i_1 \dots i_{p+1}} .$$

We are going to compare string theory S-matrix elements with field theory S-matrix elements including their coefficients, however, we are not interested in fixing the overall sign of the amplitudes. Hence, in above and in the rest of equations in this paper, we have payed no attention to the sign of equations. Replacing the trace into (10), one finds the following coupling:

$$2\beta'\mu'_p(2\pi\alpha') \text{Tr}(C_p \wedge DT) \quad (11)$$

Note that to compare the string result with the field theory result, one has to set $\alpha' = 2$ in the field theory.

3 The $T - A - C$ amplitude

The three point amplitude between one RR, one tachyon and one gauge field is given by the following correlation functions:

$$\mathcal{A}^{T,A,RR} \sim \int dx dy d^2 z \langle V_T^{(-1)}(x) V_A^{(0)}(y) V_{RR}^{(-1)}(z, \bar{z}) \rangle \quad (12)$$

where the tachyon and RR vertex operators are given in (7) and

$$V_A^{(0)}(y) = \xi_i (\partial X^i(y) + 2ik_2 \cdot \psi(y) \psi^i(y)) e^{2ik_2 X(y)} \lambda \otimes I \quad (13)$$

The σ -factor of the above S-matrix element is the same as the σ factor of the S-matrix element in the previous section, *i.e.*, $\text{Tr}(\sigma_2 I \sigma_3 \sigma_1) = 2i$. So the CP factor is $2i \text{Tr}(\lambda_1 \lambda_2)$. Using (8) and (9), one can perform the correlators above and show that the integrand is $SL(2, R)$ invariant. Fixing it as before, one finds

$$\int_{-\infty}^{\infty} dx \left(\frac{(1+x^2)^2}{16x^2} \right)^{1/4+u} \frac{2}{1+x^2} \left(\text{Tr}(P_- \not{H}_{(n)} M_p \Gamma^{ba}) k_{1a} \xi_b + \frac{1-x^2}{2x} k_1 \cdot \xi \text{Tr}(P_- \not{H}_{(n)} M_p) \right)$$

where $u = -(k_1 + k_2)^2$ and conservation of momentum along the world volume of brane, *i.e.*, $k_1^i + k_2^i + p^i = 0$, has been used. While the first term satisfies the Ward identity, the second term does not. However, the integral of the second term is zero. This indicates that there is no coupling between one C_p , one tachyon and one gauge field. This is consistent with the coupling in the first line of (5) because $\text{Tr}[A, T] = 0$. The integral of the first term is

$$\mathcal{A}^{T,A,RR} = (\pi \beta' \mu'_p) 2\pi \frac{\Gamma[-2u + 1/2]}{\Gamma[3/4 - u]^2} \text{Tr}(P_- \not{H}_{(n)} M_p \Gamma^{ba}) k_{1a} \xi_b \text{Tr}(\lambda_1 \lambda_2) . \quad (14)$$

where we have also normalized the amplitude by the factor $(-i\pi\beta\mu'_p/2)$. The trace is zero for $p \neq n+1$, and for $n+1 = p$ it is

$$\text{Tr} \left(\not{H}_{(n)} M_p(k_1 \cdot \gamma) (\xi \cdot \gamma) \right) \delta_{p,n+1} = \pm \frac{32}{(p-1)!} \epsilon^{i_1 \dots i_{p+1}} H_{i_1 \dots i_{p-1}} k_{1i_p} \xi_{i_{p+1}} \delta_{p,n+1}$$

As can be seen from the poles of the Gamma function, the above amplitude has neither massless pole nor tachyon pole. This is consistent with the WZ terms in (5). However,

there are infinite higher derivative couplings between one C_{p-2} , one tachyon and one gauge field which should be higher derivative extension of the WZ coupling $C_{p-2} \wedge F \wedge DT$. They can be read from the momentum expansion of the above S-matrix element.

The momentum expansion should be either around $(k_1 + k_2)^2 \rightarrow 0$ or around $k_1 \cdot k_2 \rightarrow 0$. The momentum expansion can be found by studying the massless/tachyon pole of field theory [13]. Since there is no massless/tachyon pole, one may conclude that the momentum expansion of this amplitude can not be found. However, the tachyon pole of the Feynman amplitude disappear because of the kinematic reason, *i.e.*, the off-shell tachyon is abelian, and the nonabelian kinetic term of tachyon in which one of the tachyons is abelian is zero. So the momentum expansion should be around $u \rightarrow -1/4$, or in terms of momentum around $k_1 \cdot k_2 \rightarrow 0$. Note that the on-shell condition implies the RR field must be non-zero, *i.e.*, $p_i p^i \rightarrow 1/4$ which is possible only for non-BPS SD-brane. Expansion of the prefactor at this point is

$$2\pi \frac{\Gamma[-2u + 1/2]}{\Gamma[3/4 - u]^2} = 2\pi \sum_{n=-1}^{\infty} b_n (u + 1/4)^{n+1} .$$

where some of the coefficients b_n are

$$\begin{aligned} b_{-1} &= 1 \\ b_0 &= 0 \\ b_1 &= \frac{\pi^2}{6} \\ b_2 &= 2\zeta(3) \\ b_3 &= \frac{19}{360}\pi^4 \\ b_4 &= \frac{1}{3}(\zeta(3)\pi^2 + 18\zeta(5)) \\ b_5 &= \frac{1}{3024}(55\pi^6 + 6048\zeta(3)^2) \end{aligned} \tag{15}$$

We will see in the next section that the whole infinite contact terms of the above momentum expansion appear in the massless pole of the S-matrix element of one C_{p-2} and three tachyons. Note that the above coefficients are exactly the same as the coefficients appear in the momentum expansion of the S-matrix element of one RR and two gauge fields [29]. Replacing the above expansion in (14), one finds that the higher derivative extension of the WZ coupling $C_{p-2} \wedge F \wedge DT$ is the following:

$$2\beta' \mu'_p (2\pi\alpha')^2 C_{p-2} \wedge \text{Tr} \left(\sum_{n=-1}^{\infty} b_n (\alpha')^{n+1} D^{a_1} \cdots D^{a_{n+1}} F \wedge D_{a_1} \cdots D_{a_{n+1}} DT \right) \tag{16}$$

which is the same as the higher derivative extension of the WZ coupling $C_{p-3} \wedge F \wedge F$ of the brane-anti-brane system. This may indicate that the higher derivative WZ couplings of non-BPS brane and the higher derivative WZ couplings of brane-anti-brane have the same structure. In the next section we calculate the S-matrix element of one RR and three tachyons.

4 The $T - T - T - C$ amplitude

The S-matrix element of one RR field and three tachyons is given by the following correlation function:

$$\mathcal{A}^{TTTC} \sim \int dx_1 dx_2 dx_3 dz d\bar{z} \langle V_T^{(-1)}(x_1) V_T^{(0)}(x_2) V_T^{(0)}(x_3) V_{RR}^{(-1)}(z, \bar{z}) \rangle$$

where we have chosen the vertex operators according to the rule that the total superghost number must be -2 . The tachyon in -1 picture and RR vertex operators are given in (7). The tachyon in 0 picture is

$$V_T^{(0)}(x) = 2ik \cdot \psi(x) e^{2ikX(x)} \lambda \otimes \sigma_1 \quad (17)$$

Introducing $x_4 \equiv z = x + iy$ and $x_5 \equiv \bar{z} = x - iy$, the scattering amplitude reduces to the following correlators:

$$\begin{aligned} \mathcal{A}^{TTTC} &\sim \int dx_1 \cdots dx_5 (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} \langle : e^{-1/2\phi(x_4)} : e^{-1/2\phi(x_5)} : e^{-\phi(x_1)} : \rangle \\ &\times \langle : e^{2ik_1 \cdot X(x_1)} : e^{2ik_2 \cdot X(x_2)} : e^{2ik_3 \cdot X(x_3)} : e^{ip \cdot X(x_4)} : e^{ip \cdot D \cdot X(x_5)} : \rangle \\ &\times \langle : S_\alpha(x_4) : S_\beta(x_5) : 2ik_2 \cdot \psi(x_2) : 2ik_3 \cdot \psi(x_3) : \rangle \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \text{Tr}(\sigma_2 \sigma_1 \sigma_1 \sigma_3 \sigma_1) \end{aligned} \quad (18)$$

The σ -factor in the above amplitude is $\text{Tr}(\sigma_2 \sigma_1 \sigma_1 \sigma_3 \sigma_1) = 2i$ for any permutation of the tachyons.

The correlators in the first and the second lines can be calculated using the propagators in (8), and the correlator in the last line can be read from (9). The result is

$$\begin{aligned} &4i \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \int dx_1 \cdots dx_5 k_{2i} k_{3j} (x_{34} x_{35})^{-1/2+2k_3 \cdot p} (x_{14} x_{15})^{-1/2+2k_1 \cdot p} (x_{24} x_{25})^{-1/2+2k_2 \cdot p} \\ &x_{45}^{p \cdot D \cdot p - 1/2} x_{23}^{4k_2 \cdot k_3} x_{12}^{4k_1 \cdot k_2} x_{13}^{4k_1 \cdot k_3} (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} \left\{ (\Gamma^{ij} C^{-1})_{\alpha\beta} + 2\eta^{ij} \frac{\text{Re}[x_{24} x_{35}]}{x_{23} x_{45}} (C^{-1})_{\alpha\beta} \right\} \end{aligned}$$

where $x_{ij} \equiv x_i - x_j$. One can show that the integrand is invariant under $\text{SL}(2, \mathbb{R})$ transformation. Fixing this symmetry as

$$x_1 = 0, \quad x_2 = 1, \quad x_3 \rightarrow \infty, \quad dx_1 dx_2 dx_3 \rightarrow x_3^2$$

One finds

$$\begin{aligned} \mathcal{A}^{TTTC} \sim & 4i \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \int dx_4 dx_5 k_{2i} k_{3j} x_{45}^{-2(t+s+u)-2} |x_4|^{2t+2s} |1-x_4|^{2u+2t} \\ & \times \left\{ (\Gamma^{ij} C^{-1})_{\alpha\beta} + 2\eta^{ij} \frac{x-1}{x_{45}} (C^{-1})_{\alpha\beta} \right\} (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} \end{aligned} \quad (19)$$

where we have also introduced the Mandelstam variables

$$s = -(k_1 + k_3)^2, \quad t = -(k_1 + k_2)^2, \quad u = -(k_2 + k_3)^2 \quad (20)$$

and used the conservation of momentum along the brane, *i.e.*, $k_1^i + k_2^i + k_3^i + p^i = 0$. They satisfy the on-shell condition

$$s + t + u = -3/4 - p_i p^i$$

Using the fact that M_p , $\mathbb{H}_{(n)}$, and Γ^{ji} are totally antisymmetric combinations of the Gamma matrices, one realizes that the first term is non-zero only for $p = n + 1$, and the last term is non-zero only for $p = n - 1$. The integral in the above equation can be written in terms of the Gamma functions, using the following identity [31]:

$$\int d^2z |1-z|^a |z|^b (z-\bar{z})^c (z+\bar{z})^d = (2i)^{c+d} \pi \frac{\Gamma(1+d+\frac{b+c}{2}) \Gamma(1+\frac{a+c}{2}) \Gamma(-1-\frac{a+b+c}{2}) \Gamma(\frac{1+c}{2})}{\Gamma(-\frac{a}{2}) \Gamma(-\frac{b}{2}) \Gamma(2+c+d+\frac{a+b}{2})}$$

for $d = 0, 1$ and arbitrary a, b, c . The region of integration is the upper half complex plane, as in our case. Using this integral, one finds that \mathcal{A}^{TTTC} is equal to

$$\frac{3\beta' \mu'_p}{2\sqrt{\pi}} \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \left[\text{Tr} \left(P_- \mathbb{H}_{(n)} M_p(k_2 \cdot \gamma)(k_3 \cdot \gamma) \right) I \delta_{p,n+1} - i \text{Tr} \left(P_- \mathbb{H}_{(n)} M_p \right) J \delta_{p,n-1} \right]$$

where we have normalized the amplitude by $-i3\beta' \mu'_p / 8\sqrt{\pi}$. There are similar terms with coefficient $\text{Tr}(\lambda_1 \lambda_3 \lambda_2)$. The extra factor of i in the second term is coming from the fact that the second term in (19) has factor $x_{45} = 2iy$. In above equation, I, J are :

$$\begin{aligned} I &= (2)^{-2(t+s+u)-1} \pi \frac{\Gamma(-u) \Gamma(-s) \Gamma(-t) \Gamma(-t-s-u-1/2)}{\Gamma(-u-t) \Gamma(-t-s) \Gamma(-s-u)} \\ J &= (2)^{-2(t+s+u)-2} \pi \frac{\Gamma(-u+1/2) \Gamma(-s+1/2) \Gamma(-t+1/2) \Gamma(-t-s-u-1)}{\Gamma(-u-t) \Gamma(-t-s) \Gamma(-s-u)} \end{aligned}$$

The traces are:

$$\begin{aligned} \text{Tr} \left(\mathbb{H}_{(n)} M_p(k_2 \cdot \gamma)(k_3 \cdot \gamma) \right) \delta_{p,n+1} &= \pm \frac{32}{n!} \epsilon^{i_1 \dots i_{p+1}} H_{i_1 \dots i_{p-1}} k_{2i_p} k_{3i_{p+1}} \delta_{p,n+1} \\ \text{Tr} \left(\mathbb{H}_{(n)} M_p \right) \delta_{p,n-1} &= \pm \frac{32}{n!} \epsilon^{i_1 \dots i_{p+1}} H_{i_1 \dots i_{p+1}} \delta_{p,n-1} \end{aligned}$$

Note that the amplitude is symmetric under interchanging s, t, u . Examining the the Feynman diagrams resulting from the WZ couplings in (5), one realizes that for the case that $p = n + 1$ there are massless poles in s -, t - and u -channels, and for the case that $p = n - 1$ there is tachyon pole in $(s + t + u)$ -channel. So the expansion should be around the following points:

$$\frac{1}{3} ([u \rightarrow 0, s, t \rightarrow -1/2] + [t \rightarrow 0, s, u \rightarrow -1/2] + [s \rightarrow 0, u, t \rightarrow -1/2]) \quad (21)$$

To see that it is momentum expansion, we write it in terms of momenta of the tachyons, *i.e.*,

$$\begin{aligned} \frac{1}{3} \Big(& [(k_2 + k_3)^2, k_1 \cdot k_3, k_1 \cdot k_2 \rightarrow 0] + [(k_1 + k_2)^2, k_1 \cdot k_3, k_2 \cdot k_3 \rightarrow 0] \\ & + [(k_1 + k_3)^2, k_1 \cdot k_2, k_2 \cdot k_3 \rightarrow 0] \Big) \end{aligned} \quad (22)$$

so the expansion should be a momentum expansion. Note again that the on-shell condition implies that the RR field must be non-zero, *i.e.*, $p_i p^i \rightarrow 1/4$ which is possible only for non-BPS SD-branes. Let us study each case separately.

4.1 $p = n + 1$ case

The electric part of the amplitude for one C_{p-2} and three tachyons is

$$\mathcal{A}^{TTTC} = \pm \frac{24\beta' \mu'_p}{\sqrt{\pi}(p-1)!} \left[\epsilon^{i_1 \dots i_{p+1}} H_{i_1 \dots i_{p-1}} k_{2i_p} k_{3i_{p+1}} \right] I \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \quad (23)$$

Note that apart from the group factor the amplitude is antisymmetric under interchanging the tachyons. So the whole amplitude is zero for abelian gauge group. To find the couplings in non-abelian theory, we expand I around (21), *i.e.*,

$$\begin{aligned} I = & \frac{2\pi\sqrt{\pi}}{3} \left(-\frac{1}{u} \sum_{n=-1}^{\infty} b_n (s+t+1)^{n+1} + \sum_{p,n,m=0}^{\infty} c_{p,n,m} u^p ((s+1/2)(t+1/2))^n (s+t+1)^m \right. \\ & - \frac{1}{t} \sum_{n=-1}^{\infty} b_n (s+u+1)^{n+1} + \sum_{p,n,m=0}^{\infty} c_{p,n,m} t^p ((s+1/2)(u+1/2))^n (s+u+1)^m \\ & \left. - \frac{1}{s} \sum_{n=-1}^{\infty} b_n (u+t+1)^{n+1} + \sum_{p,n,m=0}^{\infty} c_{p,n,m} s^p ((u+1/2)(t+1/2))^n (u+t+1)^m \right) \end{aligned}$$

where the coefficients b_n are exactly those that appear in (15) and $c_{p,0,0} = a_p$ are

$$a_0 = 4 \ln(2)$$

$$\begin{aligned}
a_1 &= \frac{\pi^2}{6} - 8 \ln(2)^2 \\
a_2 &= -\frac{2}{3} \left(\pi^2 \ln(2) - 3\zeta(3) - 16 \ln(2)^3 \right) \\
a_3 &= \frac{1}{120} \left(160\pi^2 \ln(2)^2 + 3\pi^4 - 960\zeta(3) \ln(2) - 1280 \ln(2)^4 \right) \\
a_5 &= -\frac{1}{90} \left(160\pi^2 \ln(2)^3 + 9\pi^4 \ln(2) - 1440\zeta(3) \ln(2)^2 + 30\zeta(3)\pi^2 - 768 \ln(2)^5 - 540\zeta(5) \right)
\end{aligned} \tag{24}$$

The constants $c_{p,n,m}$ for some other cases are the following:

$$\begin{aligned}
c_{0,0,2} &= \frac{2}{3}\pi^2 \ln(2), \quad c_{0,1,0} = -14\zeta(3), \quad c_{0,0,3} = 8\zeta(3) \ln(2), \\
c_{1,1,0} &= 56\zeta(3) \ln(2) - \pi^4/2, \quad c_{1,0,2} = \frac{1}{36}(\pi^4 - 48\pi^2 \ln(2)^2), \quad c_{0,1,1} = -\pi^4/2
\end{aligned}$$

It is interesting to note that the above coefficients are exactly those that appear in the momentum expansion of the S-matrix element of one C_{p-3} , two tachyons and one gauge field of brane-anti-brane system [29]. This may indicate that the higher derivative couplings of non-BPS brane and brane-anti-brane have the same structure. The massless poles and the contact terms of the S-matrix element (23) correspond to the Feynman diagrams in Fig.1.

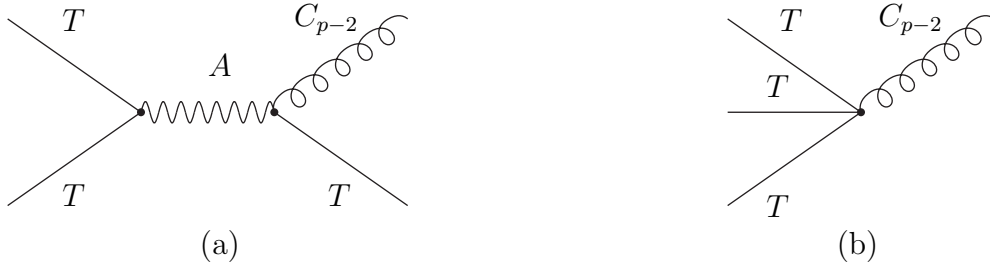


Figure 1 : a) The Feynman diagram corresponding to the amplitude (27), b) the Feynman diagram corresponding to the couplings (25).

The contact terms in (23) can be reproduced by the following couplings:

$$\begin{aligned}
& 8i\beta'\alpha'(\pi\alpha')\mu'_p \sum_{p,n,m=0}^{\infty} c_{p,n,m} \left(\frac{\alpha'}{2} \right)^p (\alpha')^{2n+m} C_{p-2} \wedge \text{Tr} \left(D^{a_1} \dots D^{a_{2n}} D^{b_1} \dots D^{b_m} DT \right. \\
& \left. \wedge (D^a D_a)^p D_{b_1} \dots D_{b_m} (D_{a_1} \dots D_{a_n} DT \wedge D_{a_{n+1}} \dots D_{a_{2n}} DT) \right)
\end{aligned} \tag{25}$$

which is the higher derivative extension of $C_{p-2} \wedge DT \wedge DT \wedge DT$. This fixes the normalization constant β' to be

$$\beta' = \frac{1}{\pi} \sqrt{\frac{6 \ln(2)}{\alpha'}} \tag{26}$$

Note that the normalization of the tachyon in the WZ terms of non-BPS brane is different from the normalization of tachyon in the WZ terms of brane-anti-brane (3). The above higher derivative extension is the same as the higher derivative extension of the coupling $C_{p-3} \wedge F \wedge DT \wedge DT^*$ of brane-anti-brane system.

The field theory has the following massless pole for $p = n + 1$:

$$\mathcal{A} = V_a^\alpha(C_{p-2}, T, A) G_{ab}^{\alpha\beta}(A) V_b^\beta(A, T, T) \quad (27)$$

where the vertices can be found from the standard nonabelian kinetic term of tachyon and from the higher derivative couplings in (16), *i.e.*,³

$$\begin{aligned} G_{ab}^{\alpha\beta}(A) &= \frac{i\delta_{ab}\delta_{\alpha\beta}}{(2\pi\alpha')^2 T_p(u)} \\ V_b^\beta(A, T, T) &= iT_p(2\pi\alpha')(k_2 - k_3)_b [\text{Tr}(\lambda_2 \lambda_3 \Lambda^\beta) - \text{Tr}(\lambda_3 \lambda_2 \Lambda^\beta)] \\ V_a^\alpha(C_{p-2}, T, A) &= 2\beta' \mu'_p (2\pi\alpha')^2 \frac{1}{(p-1)!} \epsilon_{i_1 \dots i_{p-1} a} H^{i_1 \dots i_{p-1}} k_1^{i_p} \sum_{n=-1}^{\infty} b_n (\alpha' k_1 \cdot k)^{n+1} \text{Tr}(\lambda_1 \Lambda^\alpha) \end{aligned}$$

where k is the momentum of the off-shell gauge field. The amplitude (27) becomes

$$\begin{aligned} \mathcal{A} &= 2\beta' \mu'_p (2\pi\alpha') \frac{2}{(p-1)! u} \epsilon_{i_1 \dots i_{p+1}} H^{i_1 \dots i_{p-1}} k_2^{i_p} k_3^{i_{p+1}} [\text{Tr}(\lambda_1 \lambda_2 \lambda_3) - \text{Tr}(\lambda_1 \lambda_3 \lambda_2)] \\ &\quad \times \sum_{n=-1}^{\infty} b_n \left(\frac{\alpha'}{2}\right)^{n+1} (s+t+1)^{n+1} \end{aligned}$$

this is exactly the massless pole of the string theory amplitude (23). Note that there is no left over residual contact term in comparing above amplitude with the massless pole of the string theory amplitude.

4.2 $p = n - 1$ case

Now we consider $p = n - 1$ case. The electric part is,

$$\mathcal{A}^{TTTC} = \pm \frac{24i\beta' \mu'_p}{\sqrt{\pi}(p+1)!} \left(\epsilon^{i_1 \dots i_{p+1}} H_{i_1 \dots i_{p+1}} \right) J \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \quad (28)$$

³Note that the coupling of two tachyons and one gauge field is given by the S-matrix element of two tachyons in -1 picture and one gauge field in 0 picture. Hence the internal CP factor is $\text{Tr}(I\sigma_2\sigma_2) = 2$ for the two permutation of the tachyons. One should not consider the case that the gauge field and one of the tachyon to be in -1 picture and the other tachyon to be in 0 picture, because the CP factor is $\text{Tr}(\sigma_3\sigma_2\sigma_1) = -2i$ in one ordering and is $\text{Tr}(\sigma_3\sigma_1\sigma_2) = 2i$ in other ordering of the tachyons. So the latter choice does not produce the nonabelian kinetic term of tachyon.

The amplitude is symmetric under interchanging the tachyons. So it is non-zero even for abelian case. The expansion of J around (21) is

$$\begin{aligned}
J = & \sqrt{\pi} \left(\frac{-1}{(t+s+u+1)} + \sum_{n=0}^{\infty} a_n (s+t+u+1)^n \right. \\
& + \frac{\sum_{n,m=0}^{\infty} d_{n,m} [(s+t+1)^n ((t+1/2)(s+1/2))^{m+1} + (t, s \rightarrow t, u) + (t, s \rightarrow s, u)]}{3(t+s+u+1)} \quad (29) \\
& + \sum_{p,n,m=0}^{\infty} \frac{e_{p,n,m}}{3} (s+t+u+1)^p \\
& \left. \times [(s+t+1)^n ((t+1/2)(s+1/2))^{m+1} + (t, s \rightarrow t, u) + (t, s \rightarrow s, u)] \right)
\end{aligned}$$

where the coefficients a_n in the first line are exactly those appear in (24). Some of the coefficients $d_{n,m}$ and $e_{p,n,m}$ are

$$\begin{aligned}
d_{0,0} &= -\pi^2/3, & d_{1,0} &= 8\zeta(3) \\
d_{2,0} &= -7\pi^4/45, & d_{0,1} &= \pi^4/45, & d_{3,0} &= 32\zeta(5), & d_{1,1} &= -32\zeta(5) + 8\zeta(3)\pi^2/3 \\
e_{0,0,0} &= \frac{2}{3} (2\pi^2 \ln(2) - 21\zeta(3)), & e_{1,0,0} &= \frac{1}{9} (4\pi^4 - 504\zeta(3) \ln(2) + 24\pi^2 \ln(2)^2)
\end{aligned}$$

Note that the contact terms in the last line of (29) do not have the structure of the contact terms in the first line of (29). They correspond to different couplings in field theory. Here also the above momentum expansion is the same as the momentum expansion of the S-matrix element of one C_{p-1} , two tachyons and one gauge field of the brane-anti-brane system [29]. The tachyon poles and the contact terms correspond to the Feynman diagrams in Fig.2.



Figure 2 : a) The Feynman diagram corresponding to the amplitude (32), b) the Feynman diagram corresponding to the couplings (30) and (31).

The contact terms in the second term of (29) correspond to the following couplings:

$$-24\beta'\alpha'\mu'_p \sum_{n=0}^{\infty} a_n \left(\frac{\alpha'}{2} \right)^n C_p \wedge (D^a D_a)^n [T^2 DT] \quad (30)$$

which is the higher derivative extension of the WZ term $C_p \wedge DT T^2$. The above couplings are similar to the higher derivative extension of $C_{p-1} \wedge F |T|^2$ coupling in the brane-anti-brane system. The contact terms in the last line of (29) correspond to the following couplings:

$$-8\beta'(\alpha')^2\mu'_p \sum_{p,n,m=0}^{\infty} e_{p,n,m}(\alpha')^{2m+n} \left(\frac{\alpha'}{2}\right)^p H_{p+1}(D_a D^a)^p \quad (31)$$

$$\left[D_b D_c D^{a_1} \dots D^{a_n} D_{b_1} \dots D_{b_{2m}} T D_{a_1} \dots D_{a_n} (D^b D^{b_1} \dots D^{b_m} T D^c D^{b_{m+1}} \dots D^{b_{2m}} T) \right]$$

Similar higher derivative couplings with exactly the same coefficients $e_{p,n,m}$ appear in the brane-anti-brane system [29].

The tachyon poles in (29) should be reproduced by the following amplitude:

$$\mathcal{A} = V^\alpha(C_p, T) G^{\alpha\beta}(T) V^\beta(T, T, T, T) \quad (32)$$

The propagator and vertex $V(C_p, T)$ are

$$G^{\alpha\beta}(T) = \frac{i\delta_{\alpha\beta}}{(2\pi\alpha')T_p(-k^2 - m^2)}$$

$$V^\alpha(C_p, T) = 2i\beta'\mu'_p(2\pi\alpha') \frac{1}{(p+1)!} \epsilon_{i_1 \dots i_{p+1}} H^{i_1 \dots i_{p+1}} \text{Tr}(\Lambda^\alpha)$$

To find the vertex $V^\beta(T, T, T, T)$ we need the couplings of four tachyons. The higher derivative couplings of four tachyons in which all tachyons carry σ_1 factor has been found in [30] by expanding the S-matrix element of four tachyons. Using these higher derivative couplings of four tachyons, one can not produce the tachyon pole in (29). In particular there is no tachyon pole corresponding the first term in (29), and the coefficients of the infinite tachyon poles in the second line of (29) are not proportional to $d_{n,m}$. In the next section, we find the four-tachyon couplings from the S-matrix element of four tachyons in which two tachyons are in 0 picture and the other two are in -1 picture.

4.2.1 Four tachyon couplings

The four tachyon couplings in the higher derivative field theory can be found from a momentum expansion of the S-matrix element of four tachyons in which two of them are in 0 picture and the other two are in -1 picture. There is freedom to choose each vertex operator to be in 0 picture or in -1 picture. The result is given by (see *e.g.*, [13])

$$\mathcal{A} = -12iT_p \left(A \frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-1-2t-2s)} + B \frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} + C \frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)} \right)$$

where the Mandelstam variables are

$$\begin{aligned}s &= -\alpha'(k_1 + k_2)^2/2 , \\ t &= -\alpha'(k_2 + k_3)^2/2 , \\ u &= -\alpha'(k_1 + k_3)^2/2 .\end{aligned}$$

and satisfy the constraint

$$s + t + u = -1 . \quad (33)$$

Note that our convention for the Mandelstam variables in this section is different from our convention in (20). The coefficients A, B, C are the CP factors. Using the fact that the tachyon vertex operators carry both $U(N)$ and internal σ matrices, the CP factors in this case are the following:

$$\begin{aligned}A &= \frac{1}{4} \left(\text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_4) \text{Tr}(\tau_1 \tau_2 \tau_3 \tau_4) + \text{Tr}(\lambda_1 \lambda_4 \lambda_3 \lambda_2) \text{Tr}(\tau_1 \tau_4 \tau_3 \tau_2) \right) , \\ B &= \frac{1}{4} \left(\text{Tr}(\lambda_1 \lambda_3 \lambda_4 \lambda_2) \text{Tr}(\tau_1 \tau_3 \tau_4 \tau_2) + \text{Tr}(\lambda_1 \lambda_2 \lambda_4 \lambda_3) \text{Tr}(\tau_1 \tau_2 \tau_4 \tau_3) \right) , \\ C &= \frac{1}{4} \left(\text{Tr}(\lambda_1 \lambda_4 \lambda_2 \lambda_3) \text{Tr}(\tau_1 \tau_4 \tau_2 \tau_3) + \text{Tr}(\lambda_1 \lambda_3 \lambda_2 \lambda_4) \text{Tr}(\tau_1 \tau_3 \tau_2 \tau_4) \right) .\end{aligned}$$

where λ 's are the $U(N)$ matrices and each τ is either σ_1 or σ_2 such that in the trace two of the τ 's should be σ_1 and the other two should be σ_2 . Following [13], to find the momentum expansion of this amplitude, one should first write it as $A = A_s + A_t + A_u$ where

$$\begin{aligned}A_s &= -4iT_p \left(A \frac{\Gamma(-2s)\Gamma(-2t)}{\Gamma(-1-2s-2t)} + B \frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} - C \frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)} \right) \\ A_u &= -4iT_p \left(-A \frac{\Gamma(-2s)\Gamma(-2t)}{\Gamma(-1-2s-2t)} + B \frac{\Gamma(-2u)\Gamma(-2s)}{\Gamma(-1-2u-2s)} + C \frac{\Gamma(-2u)\Gamma(-2t)}{\Gamma(-1-2u-2t)} \right) \\ A_t &= -4iT_p \left(A \frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-1-2t-2s)} - B \frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} + C \frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)} \right) \quad (34)\end{aligned}$$

The momentum expansion of the above amplitude should be around [13]

$$\begin{aligned}s - \text{channel} : & \quad \lim_{s \rightarrow 0, t, u \rightarrow -1/2} A_s \\ t - \text{channel} : & \quad \lim_{t \rightarrow 0, s, u \rightarrow -1/2} A_t \\ u - \text{channel} : & \quad \lim_{u \rightarrow 0, s, t \rightarrow -1/2} A_u\end{aligned}$$

These limits are consistent with the constraint (33). There are s-channel, t-channel and u-channel massless poles in A_s , A_t and A_u , respectively.

Now we follow [30] to expand the amplitude (34) around the above points. Using the constraint (33), we first rewrite the amplitude in the following form:

$$\begin{aligned} A_s &= 16iT_p t' u' \left(A \frac{\Gamma(2t' + 2u')\Gamma(-2t')}{\Gamma(1 + 2u')} + B \frac{\Gamma(2t' + 2u')\Gamma(-2u')}{\Gamma(1 + 2t')} + C \frac{\Gamma(-2t')\Gamma(-2u')}{\Gamma(1 - 2t' - 2u')} \right) \\ A_u &= 16iT_p t' s' \left(A \frac{\Gamma(-2s')\Gamma(-2t')}{\Gamma(1 - 2s' - 2t')} + B \frac{\Gamma(2t' + 2s')\Gamma(-2s')}{\Gamma(1 + 2t')} + C \frac{\Gamma(2t' + 2s')\Gamma(-2t')}{\Gamma(1 + 2s')} \right) \\ A_t &= 16iT_p u' s' \left(A \frac{\Gamma(2s' + 2u')\Gamma(-2s')}{\Gamma(1 + 2u')} + B \frac{\Gamma(-2u')\Gamma(-2s')}{\Gamma(1 - 2s' - 2u')} + C \frac{\Gamma(2u' + 2s')\Gamma(-2u')}{\Gamma(1 + 2s')} \right) \end{aligned}$$

where $s' = s + 1/2 = -\alpha' k_1 \cdot k_2$, $t' = t + 1/2 = -\alpha' k_2 \cdot k_3$ and $u' = u + 1/2 = -\alpha' k_1 \cdot k_3$. Note that $s + t' + u' = 0$, $u + s' + t' = 0$ and $t + s' + u' = 0$. In the first line above, u' and t' are independent, in the second line t' and s' are independent and in the last line u' and s' are independent variables. Now the field theory corresponds to expanding the above amplitude around

$$s', t', u' \rightarrow 0$$

which is a momentum expansion. The expansion of the amplitude around the above point is

$$\begin{aligned} A_s &= 16iT_p t' u' \times \left(\frac{Au' + Bt' + Cs}{4t'u's} + \sum_{n,m=0}^{\infty} [a_{n,m}(Au'^n t'^m + Bt'^n u'^m) + b_{n,m}C(u'^n t'^m + t'^n u'^m)] \right) \\ A_u &= 16iT_p t' s' \times \left(\frac{Cs' + Bt' + Au}{4t's'u} + \sum_{n,m=0}^{\infty} [a_{n,m}(Cs'^n t'^m + Bt'^n s'^m) + b_{n,m}A(s'^n t'^m + t'^n s'^m)] \right) \\ A_t &= 16iT_p s' u' \times \left(\frac{Au' + Cs' + Bt}{4s'u't} + \sum_{n,m=0}^{\infty} [a_{n,m}(Au'^n s'^m + Cs'^n u'^m) + b_{n,m}B(u'^n s'^m + s'^n u'^m)] \right) \end{aligned}$$

Some of the coefficients $a_{n,m}$ and $b_{n,m}$ are

$$\begin{aligned} a_{0,0} &= -\frac{\pi^2}{6}, \quad b_{0,0} = -\frac{\pi^2}{12} \\ a_{1,0} &= 2\zeta(3), \quad a_{0,1} = 0, \quad b_{0,1} = b_{1,0} = -\zeta(3) \end{aligned}$$

$$\begin{aligned}
a_{1,1} &= a_{0,2} = -7\pi^4/90, \quad a_{2,0} = -4\pi^4/90, \quad b_{1,1} = -\pi^4/180, \quad b_{0,2} = b_{2,0} = -\pi^4/45 \\
a_{1,2} &= a_{2,1} = 8\zeta(5) + 4\pi^2\zeta(3)/3, \quad a_{0,3} = 0, \quad a_{3,0} = 8\zeta(5), \\
b_{0,3} &= -4\zeta(5), \quad b_{1,2} = -8\zeta(5) + 2\pi^2\zeta(3)/3
\end{aligned}$$

Similar expansion has been found in [30] for the S-matrix element of two tachyons and two gauge fields, and for the S-matrix element of four σ_1 -type tachyons. Now, in the s -channel, two tachyons of one type convert to gauge field and then the gauge field converts to two tachyons of the other type⁴. The CP factor for the s -channel then becomes $A = \alpha$, $B = \beta$, and $C = -\gamma$ where

$$\begin{aligned}
\alpha &= \frac{1}{2} \left(\text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_4) + \text{Tr}(\lambda_1 \lambda_4 \lambda_3 \lambda_2) \right), \\
\beta &= \frac{1}{2} \left(\text{Tr}(\lambda_1 \lambda_3 \lambda_4 \lambda_2) + \text{Tr}(\lambda_1 \lambda_2 \lambda_4 \lambda_3) \right), \\
\gamma &= \frac{1}{2} \left(\text{Tr}(\lambda_1 \lambda_4 \lambda_2 \lambda_3) + \text{Tr}(\lambda_1 \lambda_3 \lambda_2 \lambda_4) \right).
\end{aligned}$$

Similarly, in the u -channel $C = \gamma$, $B = \beta$ and $A = -\alpha$, and in the t -channel $A = \alpha$, $C = \gamma$ and $B = -\beta$. Therefore, the amplitudes simplify to

$$\begin{aligned}
A_s &= 16iT_p t' u' \times \\
&\quad \left(\frac{\alpha u' + \beta t' - \gamma s}{4t' u' s} + \sum_{n,m=0}^{\infty} [a_{n,m}(\alpha u'^n t'^m + \beta t'^n u'^m) - b_{n,m}\gamma(u'^n t'^m + t'^n u'^m)] \right) \\
A_u &= 16iT_p t' s' \times \\
&\quad \left(\frac{\gamma s' + \beta t' - \alpha u}{4t' s' u} + \sum_{n,m=0}^{\infty} [a_{n,m}(\gamma s'^n t'^m + \beta t'^n s'^m) - b_{n,m}\alpha(s'^n t'^m + t'^n s'^m)] \right) \\
A_t &= 16iT_p s' u' \times \\
&\quad \left(\frac{\alpha u' + \gamma s' - \beta t}{4s' u' t} + \sum_{n,m=0}^{\infty} [a_{n,m}(\alpha u'^n s'^m + \gamma s'^n u'^m) - b_{n,m}\beta(u'^n s'^m + s'^n u'^m)] \right)
\end{aligned} \tag{35}$$

If one ignores the internal σ matrices, the result would be the same as above in which all terms have positive sign. In that case, the massless poles are reproduced by non-abelian kinetic terms. However, the above massless poles are reproduced by the following field theory:

$$-T_p \text{Tr} \left((\pi\alpha') m^2 T^2 + (\pi\alpha') D_a T D^a T - (\pi\alpha')^2 F_{ab} F^{ba} + T^4 \right) \tag{36}$$

⁴Note that the S-matrix element of two tachyons and one gauge field in non-PBS brane is given by either $\langle A^{(0)} T^{(-1)} T^{(-1)} \rangle$ where the internal CP factor is $\text{Tr}(I\sigma_2\sigma_2) = 2$ or $\langle A^{(-2)} T^{(0)} T^{(0)} \rangle$ where the internal CP factor is $\text{Tr}(I\sigma_1\sigma_1) = 2$.

Note that the T^4 term is absent if one ignores the σ factors [13].

The difference between the above contact terms (35) and the contact terms when there is no σ matrix is the minus sign of $b_{n,m}$. The four tachyon couplings corresponding to the latter contact terms has been found in [30]. Hence, the four tachyon couplings corresponding to the contact terms (35) are:

$$T_p(\alpha')^{2+n+m} \sum_{m,n=0}^{\infty} (\mathcal{L}_8^{nm} + \mathcal{L}_9^{nm} + \mathcal{L}_{10}^{nm} + \mathcal{L}_{11}^{nm} + \mathcal{L}_{12}^{nm}) \quad (37)$$

where

$$\begin{aligned} \mathcal{L}_8^{nm} &= m^4 \text{Tr} \left(a_{n,m} \mathcal{D}_{nm} [TTTT] - b_{n,m} \mathcal{D}'_{nm} [TTTT] + h.c. \right) \\ \mathcal{L}_9^{nm} &= m^2 \text{Tr} \left(a_{n,m} [\mathcal{D}_{nm} (TTD^\alpha T D_\alpha T) + \mathcal{D}_{nm} (D^\alpha T D_\alpha TTT)] \right. \\ &\quad \left. - b_{n,m} [\mathcal{D}'_{nm} (TD^\alpha T T D_\alpha T) + \mathcal{D}'_{nm} (D^\alpha T T D_\alpha TT)] + h.c. \right) \\ \mathcal{L}_{10}^{nm} &= -\text{Tr} \left(a_{n,m} \mathcal{D}_{nm} [D_\alpha T D_\beta T D^\beta T D^\alpha T] - b_{n,m} \mathcal{D}'_{nm} [D_\alpha T D^\beta T D_\beta T D^\alpha T] + h.c. \right) \\ \mathcal{L}_{11}^{nm} &= -\text{Tr} \left(a_{n,m} \mathcal{D}_{nm} [D_\alpha T D_\beta T D^\alpha T D^\beta T] - b_{n,m} \mathcal{D}'_{nm} [D_\beta T D^\beta T D_\alpha T D^\alpha T] + h.c. \right) \\ \mathcal{L}_{12}^{nm} &= \text{Tr} \left(a_{n,m} \mathcal{D}_{nm} [D_\alpha T D^\alpha T D_\beta T D^\beta T] - b_{n,m} \mathcal{D}'_{nm} [D_\alpha T D_\beta T D^\alpha T D^\beta T] + h.c. \right) \end{aligned}$$

where the higher derivative operators \mathcal{D}_{nm} and \mathcal{D}'_{nm} are defined as

$$\begin{aligned} \mathcal{D}_{nm}(EFGH) &\equiv D_{b_1} \cdots D_{b_m} D_{a_1} \cdots D_{a_n} E F D^{a_1} \cdots D^{a_n} G D^{b_1} \cdots D^{b_m} H \\ \mathcal{D}'_{nm}(EFGH) &\equiv D_{b_1} \cdots D_{b_m} D_{a_1} \cdots D_{a_n} E D^{a_1} \cdots D^{a_n} F G D^{b_1} \cdots D^{b_m} H \end{aligned}$$

It is not difficult to check that the infinite tower of four tachyons couplings (37) produce the contact terms in the string theory S-matrix element (35).

4.2.2 The tachyon poles in field theory

Having found the four-tachyon couplings, one can now calculate the tachyon poles in field theory (32). Since the off-shell tachyon is abelian, the vertex $V^\beta(T, T, T, T)$ of the tachyon coupling in (36) is $V^\beta(T, T, T, T) = -12iT_p(\text{Tr}(\lambda_1 \lambda_2 \lambda_3 \Lambda^\beta) + \text{Tr}(\lambda_1 \lambda_3 \lambda_2 \Lambda^\beta))$. Replacing it in (32), one finds

$$24i\beta' \mu'_p \frac{1}{(p+1)!} \epsilon_{a_0 \cdots a_p} H^{a_0 \cdots a_p} \frac{\text{Tr}(\lambda_1 \lambda_2 \lambda_3) + \text{Tr}(\lambda_1 \lambda_3 \lambda_2)}{s+t+u+1}$$

which is exactly the first term in (29). The Mandelstam variables are those defined in (20). The higher derivative vertex $V^\beta(T, T, T, T)$ can be found from the higher derivative couplings (37). The result is

$$\begin{aligned}
& 2iT_p(-\alpha')^{n+m}(a_{n,m} - b_{n,m})\text{Tr}(\lambda_1\lambda_2\lambda_3\Lambda^\beta) \left[(s+1/2)(t+1/2) \right. \\
& \times \left((k_3 \cdot k)^m (k_3 \cdot k_1)^n + (k_3 \cdot k)^n (k_3 \cdot k_1)^m + (k_2 \cdot k)^m (k_2 \cdot k_1)^n + (k_2 \cdot k)^n (k_2 \cdot k_1)^m \right. \\
& \left. + (k_3 \cdot k)^m (k_2 \cdot k)^n + (k_3 \cdot k)^n (k_2 \cdot k)^m + (k_1 \cdot k_2)^m (k_1 \cdot k_3)^n + (k_1 \cdot k_2)^n (k_1 \cdot k_3)^m \right) \\
& \left. + (1, 2, 3) \rightarrow (2, 1, 3) + (1, 2, 3) \rightarrow (3, 2, 1) \right]
\end{aligned}$$

where k^a is the momentum of the off-shell gauge field. There are similar terms which have coefficient $\text{Tr}(\lambda_1\lambda_3\lambda_2\Lambda^\beta)$. Now one can write $k_2 \cdot k = k_1 \cdot k_3 - (k^2 + m^2)/2$ and $k_3 \cdot k = k_1 \cdot k_2 - (k^2 + m^2)/2$. The $k^2 + m^2$ in the above vertex will be canceled with the $k^2 + m^2$ in the denominator of the tachyon propagator resulting a bunch of contact terms of one RR and three tachyons, *i.e.*, the diagram (b) in fig.1. They should be subtracted from the contact terms that have been extracted from the S-matrix element of one RR and three tachyons, *i.e.*, the couplings in (31). Let us at the moment ignore the contact terms and consider only the tachyon poles of the amplitude (32), *i.e.*, diagram (a) in fig.1. Replacing the above vertex in (32), one finds the following tachyon pole:

$$\begin{aligned}
& -16i\beta'\mu'_p \frac{\epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p}}{(p+1)!(s'+t'+u)} \text{Tr}(\lambda_1\lambda_2\lambda_3) \sum_{n,m=0}^{\infty} \left((a_{n,m} - b_{n,m}) \right. \\
& \times \left[s't'(t'^m s'^n + t'^n s'^m) + s'u'(u'^m s'^n + u'^n s'^m) + u't'(t'^m u'^n + t'^n u'^m) \right] \Big)
\end{aligned}$$

where $t' = t + 1/2 = -\alpha'k_1 \cdot k_2$, $u' = u + 1/2 = -\alpha'k_2 \cdot k_3$ and $s' = s + 1/2 = -\alpha'k_1 \cdot k_3$. The above amplitude can be written in the following form:

$$\begin{aligned}
& -8i\beta'\mu'_p \frac{\epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p}}{(p+1)!(s'+t'+u)} \text{Tr}(\lambda_1\lambda_2\lambda_3) \sum_{n,m=0}^{\infty} \left(d_{n,m} \right. \\
& \times \left[(t' + s')^n (t' s')^{m+1} + (t' + u')^n (t' u')^{m+1} + (u' + s')^n (u' s')^{m+1} \right] \Big)
\end{aligned}$$

similar identity has been checked explicitly in [30] for studying the massless pole of the S-matrix element of $CTTA$. The above tachyon poles are exactly equal to the tachyon poles in the second line of (29).

Finally, let us return to the contact terms that the field theory amplitude (32) produces. Using the Binomial formula, one can write the contact terms as the following:

$$8i\beta'\mu'_p \frac{\epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p}}{(p+1)!} \text{Tr}(\lambda_1\lambda_2\lambda_3) \sum_{n,m=0}^{\infty} (a_{n,m} - b_{n,m}) \left\{ \right.$$

$$\begin{aligned}
& s't' \left[\left(2 \sum_{\ell=1}^m \binom{m}{\ell} (t'^{m-\ell} s'^m + s'^{m-\ell} t'^m) + 2 \sum_{\ell=1}^n \binom{n}{\ell} (t'^{n-\ell} s'^m + s'^{n-\ell} t'^m) \right) (\alpha' k^2 + \alpha' m^2)^{\ell-1} \right. \\
& + \left. \sum_{\ell=1, j=1}^{n, m} \binom{n}{\ell} \binom{m}{j} (t'^{n-\ell} s'^{m-j} + s'^{n-\ell} t'^{m-j}) (\alpha' k^2 + \alpha' m^2)^{\ell+j-1} \right] \\
& + (s', t') \rightarrow (s', u') + (s', t') \rightarrow (t', u') \Big\}
\end{aligned}$$

There are similar couplings in the brane-anti-brane system [30]. Note that the above couplings have at least four momenta. They can be rewritten in the following form:

$$\begin{aligned}
& i8\beta' \mu'_p \frac{\epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p}}{(p+1)!} \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \sum_{p, n, m=0}^{\infty} e'_{p, n, m} (s+t+u+1)^p \\
& \left[(s+t+1)^n ((t+1/2)(s+1/2))^{m+1} + (s, t) \rightarrow (s, u) + (s, t) \rightarrow (t, u) \right]
\end{aligned}$$

where $e'_{p, n, m}$ can be written in term of $a_{n, m}$ and $b_{n, m}$. The contact terms of one RR and three tachyons of (29) have the above structure. Hence, the coefficients $e_{p, n, m}$ in the couplings (31) should be replaced by

$$e_{p, n, m} \rightarrow e_{p, n, m} - e'_{p, n, m}$$

This makes the higher derivative theory to produce the string theory S-matrix element. This ends our illustration of consistency between the momentum expansion of the S-matrix element of one RR and three tachyons around (22) and the higher derivative couplings of the field theory.

5 Discussion

In this paper we have calculated the disk level S-matrix elements of CT , CTA and $CTTT$. The on-shell conditions indicate that these S-matrix elements should be considered in the world-volume of non-BPS SD-branes. By finding their momentum expansion we have shown that the leading order terms of the expansions are consistent with the WZ terms of non-BPS SD-brane, and the non-leading terms are reproduced by some higher derivative extension of the WZ terms, *i.e.*, equations (16), (25), (30) and (31). These higher derivative terms have exactly the same coefficients as the higher derivative terms of the brane-anti-brane system [29]. These couplings have in general on-shell ambiguity. One way of fixing these on-shell ambiguity is to compare the result with the off-shell couplings in BSFT. It is shown in [20] that the WZ couplings are exact when the RR field is constant. So to write the couplings

found in [29] in such a way that have no on-shell ambiguity, one should write the couplings in the momentum space in terms of the Mandelstam variables which are written in terms of RR momentum. In fact, using the on-shell conditions, one can write the expansion in terms of the RR momentum, *e.g.*, the Mandelstam variables for the S-matrix element of $CTTA$ should be sent to $t \rightarrow -1/4$, $s \rightarrow -1/4$, $u \rightarrow 0$ [29]. Using the on-shell conditions, one can rewrite them as $(p^2 + 2k_i \cdot p) \rightarrow 0$. So in this form all the higher derivative corrections are zero for constant RR field. For non-BPS SD-branes, however, the couplings are valid when $p_i p^i \rightarrow 1/4$, so one can not compare the higher derivative couplings with the BSFT couplings.

The couplings in (36) and $a_{0,0}$, $b_{0,0}$ couplings of (37) are not consistent with the usual non-abelian tachyon DBI action. This is resulted from the fact that the tachyons should be in different pictures. So one should modify the tachyon DBI action to include the internal CP factors. To this end, we consider each field carries two matrices, the $U(N)$ matrix and the internal Pauli matrix. The Pauli matrix for the gauge field is the 2×2 identity matrix⁵ whereas for the tachyon is either σ_1 or σ_2 , *i.e.*, $T^1 = T\sigma_1$, $T^2 = T\sigma_2$. Inspired by the non-abelian DBI action [32, 33] in which there are different transverse scalar fields, one may modify the usual tachyon DBI action to the following form:

$$S_{DBI} = -\frac{T_p}{2} \int d^{p+1} \sigma \text{STr} \left(V(T^i T^i) \sqrt{1 + \frac{1}{2} [T^i, T^j] [T^j, T^i]} \right. \\ \left. \times \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a T^i (Q^{-1})^{ij} D_b T^j)} \right), \quad (38)$$

where

$$Q^{ij} = I\delta^{ij} - i[T^i, T^j] \quad (39)$$

The superscripts $i, j = 1, 2$ and there is no sum over i, j . After expanding the square roots one should specify the σ factor of each tachyon in the expansion. The tachyons of 2-tachyon terms have σ_2 , half of the tachyons of 4-tachyon terms have σ_1 and the other half have σ_2 . For the 6-tachyon terms, two of them should be along σ_2 and the others to be along σ_1 direction, and so on. The trace in above equation should be completely symmetric between all matrices of the form $F_{ab}, D_a T^i, [T^i, T^j]$ and individual T^i of the potential $V(T^i T^i)$. The trace is over the group matrices and over the σ matrices. For

⁵Note that in the S-matrix element of n gauge fields, two of them should be in -1 picture which have σ_3 and the other $n - 2$ should be in 0 picture which have identity matrix. The trace of the internal CP factor is the same as the case that all gauge fields have identity matrix.

example, $\text{Tr}[T^1, T^2][T^2, T^1] = 8\text{Tr}(T^4)$ where the trace on the right hand side is over the group matrices only. The tachyon couplings (36) and the $a_{0,0}$, $b_{0,0}$ terms in (37) are the four tachyon couplings of the above DBI action.

The potential V in above equation has the following expansion

$$V(T^i T^i) = 1 + \pi\alpha' m^2 T^i T^i + \frac{1}{2}(\pi\alpha' m^2 T^i T^i)^2 + \dots$$

where m^2 is the mass squared of tachyon, *i.e.*, $m^2 = -1/(2\alpha')$. This expansion is consistent with the potential $V(T) = e^{\pi\alpha' m^2 T^2}$ which is the tachyon potential of BSFT [16]. In above DBI action however there is a symmetric trace over the σ factors and tachyon potential has also another square root term. Performing the symmetric trace, one finds

$$\frac{1}{2}\text{STr}\left(V(T^i T^i)\sqrt{1 + [T^i, T^j][T^j, T^i]}\right) = \left(1 - \frac{\pi}{2}T^2 + \frac{\pi^2}{24}T^4 + \dots\right)(1 + T^4 + \dots)$$

The symmetric trace and the square root term do not change the sign of each term compare to the exponential potential $V(T)$. So one expects that the tachyon condensates at $T \rightarrow \infty$ and the tachyon potential becomes zero at that point.

Having found the higher derivative tachyon couplings in (37), one can find the effective couplings for slowly varying tachyon. Ignoring the second covariant derivative of the tachyon, the couplings in (37) are reduced to the four tachyon couplings of the action (38) plus the following terms:

$$T_p m^4 \alpha'^3 \zeta(3) \text{Tr}(2D_a T T D^a T T + D_a T T T D^a T + D_a T D^a T T T)$$

One may conclude from the above terms that the action (38) is not the effective action. However, the couplings in (37) have on-shell ambiguity. That is, $m^2 T \sim D D T$, so the above terms may be among the higher derivative terms that should be ignored when the second derivative of tachyon is zero. The couplings (37) appear in the tachyon poles of the S-matrix element of $CTTT$, however, as it is argued in [30], if one replaces T with DDT it does not change the tachyon poles but produces an extra contact terms. Hence, this on-shell ambiguity may be resolved by studying the S-matrix element of four tachyons and one gauge field, in which the couplings (37) appear in tachyon poles as well as the contact terms of this amplitude. This S-matrix element has been found in [14].

We have considered the S-matrix element in section 4.2.1 as $\langle T^{(0)} T^{(0)} T^{(-1)} T^{(-1)} \rangle$. On the other hand one can write the amplitude as $\langle T^{(0)} T^{(0)} T^{(0)} T^{(-2)} \rangle$ in which the internal CP factor is $\text{Tr}(\sigma_1 \sigma_1 \sigma_1 \sigma_1) = 2$. The amplitude in the latter case is given by

$$\mathcal{A} = -4iT_p \left(\alpha \frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-1-2t-2s)} + \beta \frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} + \gamma \frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)} \right) \quad (40)$$

One expects that the amplitude in section 4.2.1 reduces to the above amplitude after performing the trace over the internal CP matrices. In fact using the observation that in the s-channel $A = \alpha$, $B = \beta$, $C = -\gamma$ and similarly for the other channels, one finds that the amplitude in (34) simplifies to $A_s = A_t = A_u$ and equal to the above amplitude. However, the momentum expansion is different from the momentum expansion considered in [30], *i.e.*, A_s, A_t, A_u here are different from A_s, A_t, A_u in [30]. That is the reason that the tachyon pole in \mathcal{CTTT} does not produce by the tachyon couplings found in [30]. The interesting point that we have considered $\langle T^{(0)}T^{(0)}T^{(-1)}T^{(-1)} \rangle$ in section 4.2.1 is that the momentum expansion of (34) is similar to the momentum expansion of the massless transverse scalars, hence, the four tachyon coupling can be written in the tachyon DBI form (38) in which there is a symmetric trace over $U(N)$ and over the internal CP matrices.

Therefore, there are two different expansion for the S-matrix element $\langle T^{(0)}T^{(0)}T^{(0)}T^{(-2)} \rangle$. One is the expansion considered in [30] which is consistent with the usual tachyon DBI action and the other one which is consistent with (38). Our on-shell calculation is valid only for non-BPS SD-branes. On the other hand, it is expected that the usual tachyon DBI action should describe the non-BPS D-branes. So one may conclude that the DBI part of the effective action of non-BPS SD-brane and non-BPS D-branes are not equal. To check explicitly which tachyon DBI action corresponds to non-BPS D-branes, one needs to study the S-matrix element of four tachyons and one RR field on the world volume of D-brane-anti-D-brane in which the tachyon pole is reproduced by the \mathcal{CTT} and \mathcal{TTTT} couplings. One of the above two expansions for \mathcal{TTTT} produces the tachyon poles, hence, it fixes the tachyon action. It would be interesting to perform this calculation.

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